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Are Casimir forces conservative?

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Abstract

The belief among investigators of the Casimir effect is that it is conservative and that the two interpretations as to the source of energy behind the Casimir effect are equivalent. In the first, the energy source is considered to be the zero-point fields in the vacuum of space between the plates. In the second, the source is considered to come from the potential energy of atoms in the bulk matter making up the plates. The bulk matter interpretation will be trivially conservative because the plates comprise a closed system. The vacuum energy interpretation is not as clear. If the zero-point fields between the plates are static, the system will be closed and the forces will be conservative. However the fields may be a dynamic steady state in which case the system could be open and the forces not conservative. This paper examines a plausible but speculative extension of the vacuum centric proximity force approximation that incorporates the local geometry of non-parallel plates. This inclusion eliminates problems of local scale nonsensical distances between plates as well as some absurd results but introduces non-conservative forces. While only an experiment or more fundamental analytic approach can determine whether Casimir forces are conservative, the purpose of this paper is to merely raise the question.

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1. Introduction

The belief among investigators of the Casimir Effect [1] is that it is conservative, meaning that forces produced between two closely spaced neutral conducting plates of any shape placed in a vacuum will be equal and opposite. There are two interpretations as to the source of energy behind this effect [2,3]. In

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the first, the energy source is considered to be the zero-point fields in the vacuum of space between the plates. In this interpretation the reflecting facing surface areas of the plates are all that count. The overall volume and three-dimensional shape of the plates do not enter into the calculations. What is calculated is an energy per unit of surface area which by way of the gradient of that energy, forces per unit area can be calculated.

The second interpretation as to the source of energy is that it results from the interaction energy of polarizable atoms in the bulk matter of the plates. Here the interaction of pairs of atoms within the volume of the plates (where one atom is in one plate and the other atom is in the other) are what count. The simplest approximation for calculating energy and forces between plates is to add up the energy contributions from each pair of atoms. Strictly speaking the total energy resulting from the pairs of atoms are somewhat less than the sum of the energy of the pairs because atoms inside of the plates are shielded by those closer to the facing surfaces [4].

Even though the physical descriptions of these interpretations are quite different they are none-the-less considered to be equivalent and both interpretations are believed to predict conservative forces. From thermodynamic considerations the forces originating from the bulk matter of the plates will be trivially conservative because it is a closed system. There are a finite number of atoms in each of the plates and a finite number of pairs of atoms between plates. Whereas the number of atoms in each plate may be different, the number of pairs of atoms is the same irrespective of this difference. Thus the total energy on each plate will be the same and since there is no net gradient between the plates there is no net force on the plate pair system and the resulting forces between plates will be conservative.

The energy from a vacuum interpretation is not as clear. If the zero-point fields between the plates are static, the system will be closed and the forces will be conservative. However the fields maybe in a dynamic steady state and only appear to be static. Indeed, virtual photons making up the fields are believed to randomly pop into and out of existence. In such case the system could be open and the forces may not be conservative.

The Casimir energy per unit area e_{pp} derived from the zero point fields in a vacuum for two perfectly conducting parallel plates is:

$$e_{pp}(z) = -\frac{\hbar c \pi^2}{720 z^3} \quad (1)$$

where z is the distance between plates. The normal force per unit area on the plates is:

$$f(z) = -\frac{\partial e(z)}{\partial z} = -\frac{\partial}{\partial z} \left(-\frac{\hbar c \pi^2}{720 z^3} \right) = -\frac{\hbar c \pi^2}{240 z^4} \quad (2)$$

The force is attractive.

In a paper by Milonni, Cook and Goggin [5] the Casimir forces are calculated explicitly from the radiation pressure of the vacuum. The result for parallel perfectly conducting plates is the classical Casimir formulation of the force per unit area as given in Eq. 2.

The simplest approximation for calculating Casimir forces between plates that are not parallel but that none-the-less have certain geometric constraints is the proximity force approximation (PFA). It consists of dividing up the plates into a set of parallel plates. The energy per unit area for each of the parallel plates is added together and then divided by the total flat parallel plate area to produce an average energy per unit area for the surface. Normal and lateral forces are obtained from the energy gradient in the traditional way. Normal forces thus obtained can be shown to be trivially conservative. The lateral forces (for plate geometries where lateral forces are relevant, such as sinusoidally corrugated plates) are not as easily shown to be conservative but can none-the-less be demonstrated to be conservative.

The proximity force approximation ignores the local geometry of non-parallel plates, which leads to the use of nonsensical distances between plates at a local scale as well as some absurd results. When the

local geometry is included as a working hypothesis in an extension of the proximity force approximation, the difficulties with the local geometry disappear but the forces appear to be non-conservative. Since the extension is not derived from first principles, this approach is speculative. There are however a number of constraints that this extension must abide by, as will be shown later.

This apparent anomalous non-conservative force could possibly be resolved by the use of newer “exact” methods [6] of calculating Casimir forces or by experiment. This paper will propose a conceptually simple experiment that will test the conservativeness of Casimir forces as well as indirectly test the equivalence between the two interpretations for the source of Casimir energy.

2. Pair Wise Summation Approximation

2.1. Trivially conservative

The pair wise summation (PWS) approximation is the most straightforward bulk matter energy source interpretation. It derives from the interaction energy of pairs of atoms in bulk matter and can be shown to produce results that are trivially conservative.

The interaction energy between a pair of polarizable atoms from two plates labeled A and B , can be found using the Casimir-Polder equation [4, 7].

$$u(r_{ij}) = -\frac{\hbar c}{4\pi \cdot r_{ij}^7} \left[23 \cdot (\alpha_E^A \alpha_E^B + \alpha_M^A \alpha_M^B) - 7 \cdot (\alpha_E^A \alpha_M^B + \alpha_M^A \alpha_E^B) \right] \quad (3)$$

The variable r_{ij} is the distance between atoms i and j in plates A and B respectively. The variables α_E and α_M are the electrostatic and magnetic polarizability of the atoms in the plates. At the scale of the plates the atoms can be considered point objects and the interaction energy between atoms i and j will be the same at atom i and atom j . Indeed, the Casimir-Polder equation does not distinguish between atoms i and j . The force on atom i resulting from atom j will be equal and opposite along r_{ij} to the force on atom j resulting from atom i .

Each atom i in plate A will interact with each atom j in plate B resulting in there being ij pairs of interactions. Moreover each pair of forces resulting from the interaction of a pair of atoms from plates A and B can be decomposed into normal and lateral components. The normal and lateral components of each atom pair will be conservative and if the normal and lateral components for each pair are chosen along a consistent set of axes the normal and lateral forces on each plate will be trivially conservative.

2.2. Force difference between PWS and Casimir calculation for parallel plates

The force originating from the zero-point fields between a set of parallel plates having area A is calculated by multiplying the force per unit area obtained in Eq. 2 by the area. If this force is compared to the force originating from bulk matter obtained by summing up forces between pairs of atoms between the same two plates starting from the Casimir-Polder interactive energy Eq.3, one finds that the PWS force can be an order of magnitude or more smaller than the forces calculated using Eq. 2, depending upon the distance between plates. These results are well known and are accounted for by a normalization or calibration process [4, 7, 8].

It is also known that the energy and forces between pairs of atoms in the two plates are not additive because of the screening of more distant atoms by closer ones. The implication of this effect is that the

PWS calculated forces are even smaller relative to the energy from vacuum originating calculation than the simple additive PWS approximation would indicate.

Another difference between the two calculations is if the plates are not sufficiently thick then thickness of the plates is another factor reducing the PWS calculated force relative to the parallel plates Casimir force. One could imagine that if the plates are not sufficiently thick, radiation could leak through reducing the subsequent forces between the plates. There are in fact two corrections that are applied to the zero-point fields originating forces as a result of temperature and finite conductivity of the plates. For plate separation distances much less than a micron, the thermal corrections can be ignored since infrared radiation is already excluded from the Casimir cavity. The corrections resulting from finite conductivity however are interesting in that the correction involves a plasma wavelength or plasma frequency but does not involve the thickness of the plate. Thus the zero-point fields interpretation of energy origin does not involve plate thickness in its calculation of forces or in its correction for radiation leakage while the bulk matter interpretation of energy source inherently does. Though these two interpretations are generally considered equivalent the fact that 1) the PWS calculation of forces between parallel plates is much less than the classical Casimir calculation of forces between parallel plates, 2) screening of more distant atoms by closer atoms in the bulk matter calculation makes that difference even greater and 3) the fact that the zero-point vacuum interpretation doesn't involve the plate thickness in its force calculation and corrections whereas the plate thickness is inherent in the bulk matter interpretation leads one to question just how equivalent these two interpretations are.

3. Proximity Force Approximation

The PFA is the simplest and most straightforward method of calculating Casimir forces between near parallel plates that assumes that energy comes from the vacuum of space. It derives from the classical parallel plates calculation for the Casimir forces which in turn is derived from the zero-point quantum fluctuations in the vacuum of space. Whereas the PWS approximation can be applied to all manner of shapes the PFA is restricted to plate surfaces that are near to parallel [7]. The approximation consists of dividing up the plates into a set of parallel plates as shown in Fig.1.

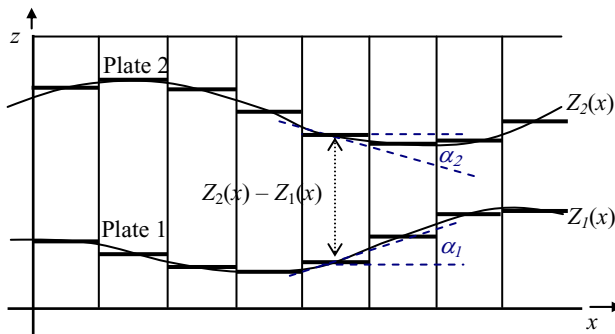


Fig. 1. Dividing Casimir plates into a set of parallel plates for the proximity force approximation

The energy per unit area for each of the parallel plates is added together and then divided by the total flat parallel plate area to produce an average energy per unit area for the surface. The total flat parallel plate area is the projection of the plate surface area onto the xy plane for a suitably chosen coordinate system. Normal and lateral forces are obtained from the energy gradient in the traditional way.

3.1. Normal forces trivially conservative

Using Fig. 1 for simplicity, functions $Z_1(x)$ and $Z_2(x)$ represent an edge on view of three dimensional surfaces defined in x, z coordinates and independent in y . The PFA energy per unit area where area is in the x, y plane is then:

$$\frac{dE(x)}{dA_{xy}} = e(x) = -\frac{\hbar c \pi^2}{720} \cdot \frac{1}{(Z_2(x) - Z_1(x))^3} \quad (4)$$

The force per unit area where area is again in the x, y plane is then:

$$\frac{d\vec{F}}{dA_{xy}} = \vec{f} = -\frac{\partial e}{\partial x} \hat{i} - \frac{\partial e}{\partial z} \hat{k} \quad (5)$$

The normal components of force on plates 1 and 2 are obtained by assuming that functions Z_1 and Z_2 are the independent variables z_1 and z_2 . Then for plate 1 the normal component of force per unit of area will be:

$$f_{z1} = -\frac{\partial e}{\partial z_1} = -\frac{\partial}{\partial z_1} \left(-\frac{\hbar c \pi^2}{720} \cdot \frac{1}{(z_2 - z_1)^3} \right) = \frac{\hbar c \pi^2}{720} \cdot \left(\frac{-3 \cdot (-1)}{(z_2 - z_1)^4} \right) = \frac{\hbar c \pi^2}{240} \cdot \frac{1}{(Z_2 - Z_1)^4} \quad (6)$$

Similarly for plate 2 the normal component of force per unit area will be:

$$f_{z2} = -\frac{\partial e}{\partial z_2} = -\frac{\partial}{\partial z_2} \left(-\frac{\hbar c \pi^2}{720} \cdot \frac{1}{(z_2 - z_1)^3} \right) = \frac{\hbar c \pi^2}{720} \cdot \left(\frac{-3}{(z_2 - z_1)^4} \right) = -\frac{\hbar c \pi^2}{240} \cdot \frac{1}{(Z_2 - Z_1)^4} = -f_{z1} \quad (7)$$

Notice that f_{z1} on the bottom Z_1 plate is in the positive direction and f_{z2} on the top Z_2 plate is in the negative direction. Thus the forces are attractive and since $f_{z1} = -f_{z2}$ for all parts of area A_{xy} , the normal forces are trivially conservative.

3.2. Conservativeness of lateral forces

The lateral force components also turn out to be conservative but can't be shown to be as trivially so. Starting with the energy per unit area Eq. 4 and assuming that Z_1 is a function of x_1 and Z_2 is a function of x_2 one obtains the lateral forces per unit area A_{xy} on the plate 1:

$$f_{x1} = -\frac{\partial e}{\partial x_1} = -\frac{\partial}{\partial x_1} \left(-\frac{\hbar c \pi^2}{720} \cdot \frac{1}{(Z_2(x_2) - Z_1(x_1))^3} \right) = \frac{\hbar c \pi^2}{720} \cdot \left(\frac{-3 \cdot \left(-\frac{\partial Z_1(x_1)}{\partial x_1} \right)}{(Z_2(x_2) - Z_1(x_1))^4} \right) = \frac{\hbar c \pi^2}{240} \cdot \frac{\frac{\partial Z_1(x)}{\partial x}}{(Z_2(x) - Z_1(x))^4} \quad (8)$$

Similarly for plate 2:

$$f_{x2} = -\frac{\partial e}{\partial x_2} = -\frac{\partial}{\partial x_2} \left(-\frac{\hbar c \pi^2}{720} \cdot \frac{1}{(Z_2(x_2) - Z_1(x_1))^3} \right) = \frac{\hbar c \pi^2}{720} \cdot \left(\frac{-3 \cdot \left(\frac{\partial Z_2(x_2)}{\partial x_2} \right)}{(Z_2(x_2) - Z_1(x_1))^4} \right) = -\frac{\hbar c \pi^2}{240} \cdot \frac{\frac{\partial Z_2(x)}{\partial x}}{(Z_2(x) - Z_1(x))^4} \quad (9)$$

The magnitude of the lateral forces per unit x, y area are obviously not equal, however from Eq. 4 the energy per unit x, y area on each plate are equal for the same point x, y . The x, y areas for both plates are also equal (both plates totally overlap). Thus the total energy on both plates is equal. With no energy gradient between the plates there are no net forces on the plate system for both normal and lateral forces and any forces between the plates are conservative. Moreover, the total lateral forces on plates 1 and 2 can be demonstrated numerically to be equal and opposite of each other even when functions Z_1 and Z_2 are different.

3.3. Proximity force approximation local geometry problem

One problem with the PFA can be seen in Fig. 2, which shows the local geometry for a section of plates with surfaces Z_1 and Z_2 seen edge on.

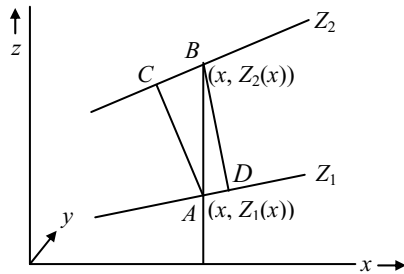


Fig.2. Problem with local geometry for a section of plates

For the cross section portion shown in Fig. 2 the line segment AB is used to calculate the local energy per unit area and force per unit area using the PFA, even though point C on Z_2 is closer to A than point B on Z_2 . Also point D on Z_1 is closer to B than point A on Z_1 . A shorter distance would mean a larger energy per unit plate area and force per unit plate area in the vicinity.

3.4. Proximity force approximation infinity problem

Subsequent sections may use the following nomenclature.

Nomenclature

Symbol	Value	Description
z_0	233 nm	Nominal distance between plates
ϕ	varies	Phase angle
λ	1.2 μm	Corrugation wavelength
A_1	59 nm	Corrugation amplitude for Z_1 plate
A_2	8 nm (unless indicated)	Corrugation amplitude for Z_2 plate
R	100 μm	Radius of corrugated sphere

Another problem with the PFA has to do with lateral force components that approach infinity instead of rightly going to zero. The problem can be illustrated with the Chen / Mohideen experiment [9] in which Casimir forces were measured between a corrugated sphere and a corrugated flat plate. In the paper a circuitous route was taken to produce an analytic result for lateral forces. The average energy per unit area for two corrugated flat plates (e_{cor}) was used to calculate the total normal force between a corrugated sphere and corrugated flat plate (F_{norm_cor}) by substituting e_{cor} for the parallel plates energy per unit area (e_{pp}) in the plane sphere equation $F_{norm}(a) = 2\pi R * e_{pp}(a)$ where F_{norm} is the total normal force between sphere and plane, R is the radius of the sphere and a is the shortest distance between plane and sphere ($R \gg a$) [9, 10, 11]. The energy between a corrugated sphere and flat plate are obtained by integrating the normal force with respect to separation. The lateral force is then obtained by differentiating with respect to phase angle ϕ (not the x displacement). This procedure removes the infinity problem. However it is an approximation of an approximation.

Fig.3 shows the geometry of a conducting corrugated spherical plate with a conducting corrugated flat plate that was used for the Chen / Mohideen analytic calculation and is also suitable for a direct numerical calculation of lateral Casimir forces using the PFA.

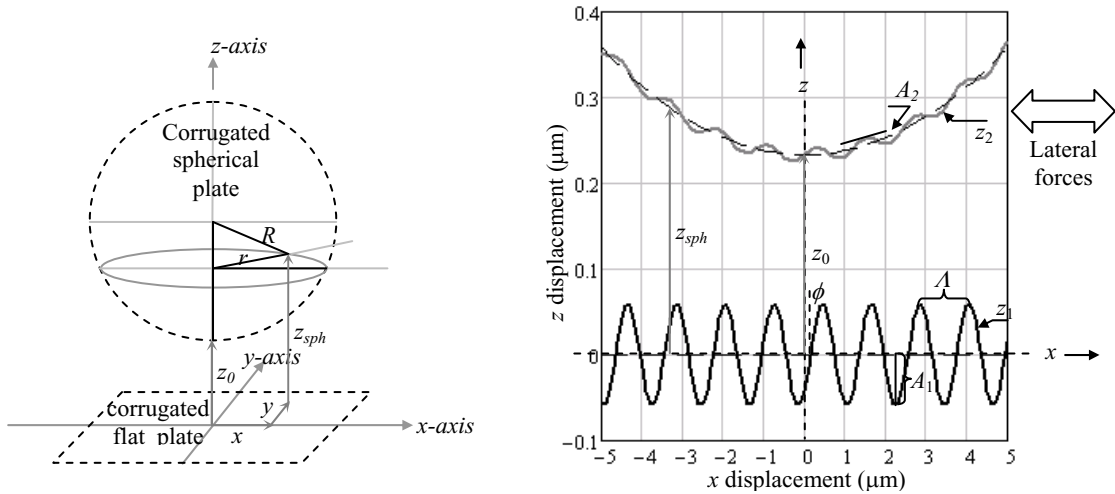


Fig. 3. Geometry for corrugated spherical plate with corrugated flat plate with overview shown on left and detailed cross section shown on right. Not to scale.

In order to use Eq. 8 and 9 to calculate the lateral forces one must first obtain functions Z_1 and Z_2 .

$$Z_1 = A_1 \cdot \sin\left(\frac{2\pi \cdot x}{\Lambda} + \phi\right)$$

$$Z_2 = z_{sph} + A_2 \cdot \sin\left(\frac{2\pi \cdot x}{\Lambda}\right)$$

$$z_{sph} = z_0 + R - \sqrt{R^2 - x^2 - y^2}$$

(10)

The infinity problem arises when one takes the partial derivative of Z_2 with respect to x as one needs to do in order to use Eq. 9.

$$\frac{\partial Z_2}{\partial x} = \frac{x}{\sqrt{R^2 - x^2 - y^2}} + \frac{2\pi}{\Lambda} \cdot A_2 \cdot \cos\left(\frac{2\pi \cdot x}{\Lambda}\right) \quad (11)$$

To find the total lateral force on the corrugated spherical plate to maximal effect, one needs to integrate Eq. 9 with Eq.11 substituted in it over the projected area of the sphere onto the x, y plane. In other words out to $R = (x^2 + y^2)^{1/2}$, the radius of the sphere. However if one attempts to do that the force per unit area described in Eq. 9 becomes infinite when it should be approaching zero. In actuality it is not necessary to integrate out to the radius R because $R \gg z_0$ and contributions to Casimir forces from areas close to the circumference of the sphere, both lateral and normal, are miniscule.

4. Extended proximity force approximation

4.1. Deriving an extended proximity force approximation energy equation

The previous discussion provides some of the motivation for seeking an extension of the PFA. Because the extension being proposed here does not start at a fundamental level it is by its very nature speculative. None-the-less, one starts by observing that when plate surfaces Z_1 and Z_2 are both parallel with the x, y plane then the line AB of Fig. 2 is both perpendicular to Z_1 at A and Z_2 at B and AB is the shortest distance between Z_1 and Z_2 . When surfaces Z_1 and Z_2 are not parallel the situation is not as simple as can be seen in Fig. 4 where surfaces Z_1 and Z_2 are again seen edge on. For simplicity only two-dimensional geometries will be considered.

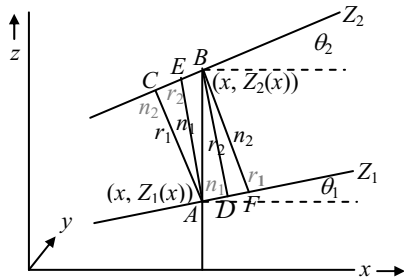


Fig. 4. Geometry of an extended proximity force approximation

Point B on surface Z_2 is directly over point A on surface Z_1 (in shown coordinate system) but point C on Z_2 is the closest point to A while point E is the perpendicular or normal projection point from A . Similarly point D on surface Z_1 is the closest point to B on Z_2 while point F is the perpendicular or normal projection point from B . In Fig. 4 the distance AC is labeled r_1 with respect to point A and distance AE is labeled n_1 with respect the same point. Similarly distance BD is labeled r_2 with respect to point B and distance BF is labeled n_2 with respect to the same point. In a sense the r_1, r_2, n_1 and n_2 distances are more natural than the PFA's AB segment. The lighter shaded r_1, r_2, n_1 and n_2 are with reference to points other than A and B .

The Casimir energy equation used for the PFA, Eq. 4, had a differential for area dA_{xy} that was in the x, y plane. In the extended PFA it is desired and is more natural to have the area differential with respect to

the plate surface. Thus the area differential for the Z_1 surface could be called dA_{s1} (the independent y coordinate is dropped from the notation for simplicity) and the area differential with respect to the Z_2 surface could be called dA_{s2} .

Eq. 4 assumes that the distance between plates is $Z_2(x) - Z_1(x)$. But this distance has been replaced by distances $r_1(x)$ and $n_1(x)$ for point A on Z_1 and by $r_2(x)$ and $n_2(x)$ for the point B on Z_2 . Next it assumes that left and right hand sides of the equation are oriented in the same direction. However dA_{s1} and dA_{s2} can be considered vectors since $d\vec{A}_{s1} = d\vec{s}_1 \times d\vec{y}_1$ is normal to the plane s_1, y_1 and $d\vec{A}_{s2} = d\vec{s}_2 \times d\vec{y}_2$ is normal to the plane s_2, y_2 at the micro level. Proceeding with this logic the left hand side of the equation becomes

$$\frac{dE_1}{dA_{s1}} = \frac{dE_1 \cdot d\vec{A}_{s1}}{d\vec{A}_{s1} \circ d\vec{A}_{s1}} = \frac{dE_1 \cdot d\vec{A}_{s1}}{|d\vec{A}_{s1}| \cdot |d\vec{A}_{s1}|} = \frac{dE_1}{dA_{s1}} \cdot \hat{n}_1 \text{ where } \hat{n}_1 \text{ is a unit vector normal to the plate area } dA_{s1} \text{ and}$$

$$\frac{dE_2}{dA_{s2}} = \frac{dE_2 \cdot d\vec{A}_{s2}}{d\vec{A}_{s2} \circ d\vec{A}_{s2}} = \frac{dE_2 \cdot d\vec{A}_{s2}}{|d\vec{A}_{s2}| \cdot |d\vec{A}_{s2}|} = \frac{dE_2}{dA_{s2}} \cdot \hat{n}_2 \text{ where } \hat{n}_2 \text{ is a unit vector normal to the plate area } dA_{s2}.$$

Similarly for the right side of Eq. 4,

$$-\frac{\hbar \cdot c \cdot \pi^2}{720} \cdot \frac{1}{2} \left(\frac{1}{r_1^2 \cdot \vec{r}_1} + \frac{1}{n_1^2 \cdot \vec{n}_1} \right) = -\frac{\hbar \cdot c \cdot \pi^2}{1440} \left(\frac{\vec{r}_1}{r_1^2 \cdot \vec{r}_1 \circ \vec{r}_1} + \frac{\vec{n}_1}{n_1^2 \cdot \vec{n}_1 \circ \vec{n}_1} \right) = -\frac{\hbar \cdot c \cdot \pi^2}{1440} \cdot \left(\frac{\hat{r}_1}{r_1^3} + \frac{\hat{n}_1}{n_1^3} \right),$$

The variables \hat{r}_1 and \hat{n}_1 are unit vectors along r_1 and n_1 respectively. By putting these left and right parts together and dot multiplying both sides by the unit vector \hat{n}_1 one then gets a new energy equation for plate Z_1 .

$$\frac{dE_1}{dA_{s1}} = -\frac{\hbar c \pi^2}{1440} \cdot \left(\frac{\hat{r}_1 \circ \hat{n}_1}{r_1^3} + \frac{1}{n_1^3} \right) \quad (12)$$

Because r_1 , n_1 and s_1 are not equal to r_2 , n_2 and s_2 there is a separate energy equation for plate Z_2 derivable from Eq. 12 by substituting 2 for 1 in the appropriate subscripts. One also notes that when the plates Z_1 and Z_2 become flat and parallel the dot product becomes 1, r_1 and n_1 become $z = Z_2 - Z_1$, dA_{s1} becomes simply dA_x and Eq. 12 becomes Eq. 4.

It is desirable to put Eq. 12 in terms of x_1 and x_2 . Accordingly, the dot product of the unit vectors of r_1 and n_1 can be seen from examining Fig. 4 to be $\cos(\theta_2 - \theta_1)$ (angle $CAB = \theta_2$ and angle $EAB = \theta_1$). Further, it may be observed that $dZ_1/dx_1 = \tan \theta_1$ and $dZ_2/dx_2 = \tan \theta_2$. By using appropriate trigonometric identities it is found that:

$$\hat{r}_1 \circ \hat{n}_1 = \frac{1 + \frac{dZ_1}{dx_1} \cdot \frac{dZ_2}{dx_2}}{\sqrt{\left(\frac{dZ_1}{dx_1}\right)^2 + 1} \cdot \sqrt{\left(\frac{dZ_2}{dx_2}\right)^2 + 1}} = \hat{r}_2 \circ \hat{n}_2 \quad (13)$$

The area differential dA_{s1} in the s_1, y_1 plane on the surface Z_1 is converted to an area differential dAx_1 in the x_1, y_1 plane by the following:

$$dA_{s1} = \sqrt{\left(\frac{dZ_1}{dx_1}\right)^2 + 1} \cdot dAx_1 \quad (14)$$

Similarly the transformation of the differential dA_{s2} to dAx_2 is accomplished by substituting 2 for 1 in the subscripts for s_1, x_1 and Z_1 in Eq. 14. Lastly the distances $r_1(x_1)$, $n_1(x_1)$, $r_2(x_2)$ and $n_2(x_2)$ are found by a set of implicit equations as follows:

$$\begin{aligned}
r_1(x_1) &= [(X_{2r}(x_1) - x_1)^2 + (Z_2(X_{2r}(x_1)) - Z_1(x_1))^2]^{1/2} \\
n_1(x_1) &= [(X_{2n}(x_1) - x_1)^2 + (Z_2(X_{2n}(x_1)) - Z_1(x_1))^2]^{1/2} \\
r_2(x_2) &= [(x_2 - X_{1r}(x_2))^2 + (Z_2(x_2) - Z_1(X_{1r}(x_2)))^2]^{1/2} \\
n_2(x_2) &= [(x_2 - X_{1n}(x_2))^2 + (Z_2(x_2) - Z_1(X_{1n}(x_2)))^2]^{1/2}
\end{aligned} \tag{15}$$

The functions X_{1r} , X_{1n} , X_{2r} and X_{2n} can be found by noting which line segments are perpendicular to which surface and then solving the resulting implicit equations.

4.2. Comparing energy in PFA with extended PFA for a set of corrugated flat plates

It is desirable to compare the calculated energy from the PFA in Eq. 4 with the calculated energy from the extended PFA using Eq. 12-15. The appropriate Z_1 and Z_2 functions for a set of corrugated flat plates can be obtained from Eq. 10 describing the corrugated sphere and corrugated flat plate by substituting z_0 for z_{sph} . Other variables are as indicated in Fig. 3. The values used in the calculations are those used in the Chen / Mohideen experiment shown in nomenclature with the added stipulation that phase angle $\phi = 0$. Calculated energy per unit x, y area obtained by using the PFA is compared in Fig. 5 with the energy per unit x, y area on the Z_1 and Z_2 plates obtained by using the extended PFA. For the extended PFA, Fig. 5 also shows the difference in energy per unit x, y area between the Z_1 and Z_2 plates. The difference in energy per unit area for the regular PFA is zero as was mentioned previously.

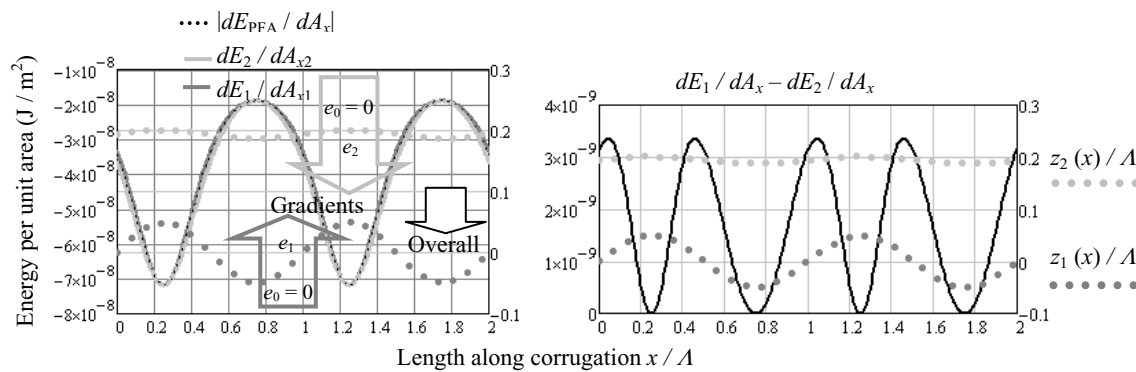


Fig. 5. Comparing energy per unit area calculated using PFA with extended PFA on Z_1 and Z_2 plates for a corrugated set of flat plates

The extended PFA compares well with the regular PFA except for the small energy gradient between the Z_1 and Z_2 plates. To be noted, the energy per unit x, y area difference is greater than or equal to zero for an entire corrugated wavelength. Also it is greatest when the difference in slopes of Z_1 and Z_2 are greatest and zero when the slopes are zero. The overall gradient of the corrugated plates system is from the plate with the lesser amplitude to that with the greater.

In Fig. 6 the energy per unit x, y area is integrated over the area of the Z_1 and Z_2 plates, which for convenience have been set to 1 cm x 1 cm. The total plate energy obtained by the PFA and the total plate energies obtained by the extended PFA for each of the Z_1 and Z_2 plates is then plotted against the phase angle between the corrugations in the Z_1 and Z_2 plates.

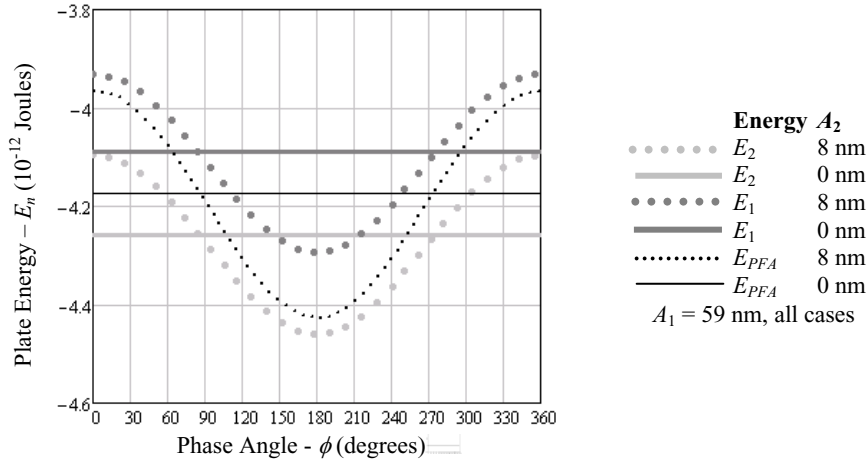


Fig. 6. Total calculated energy on 1 cm x 1 cm plates using PFA and extended PFA

4.3. Deriving extended PFA force equations

The extended PFA force equations are derived in the usual way from the extended PFA energy equation, Eq. 12.

$$\frac{d\vec{F}_1}{dA_{s1}} = \frac{d}{dA_{s1}}(-\nabla E_1) = -\nabla \frac{dE_1}{dA_{s1}} = -\nabla \left(-\frac{\hbar c \pi^2}{1440} \cdot \left(\frac{\hat{r}_1 \circ \hat{n}_1}{r_1^3} + \frac{1}{n_1^3} \right) \right) \quad (16)$$

Taking the gradient in two dimensions on Eq. 16 leads to:

$$\frac{d\vec{F}_1}{dA_{s1}} = -\frac{\hbar c \pi^2}{480} \left[\left(\frac{\hat{r}_1 \circ \hat{n}_1}{r_1^4} \cdot \frac{\partial r_1}{\partial x_1} + \frac{1}{n_1^4} \cdot \frac{\partial n_1}{\partial x_1} \right) \hat{i} + \left(\frac{\hat{r}_1 \circ \hat{n}_1}{r_1^4} \cdot \frac{\partial r_1}{\partial z_1} + \frac{1}{n_1^4} \cdot \frac{\partial n_1}{\partial z_1} \right) \hat{k} \right] \quad (17)$$

One is interested in obtaining the force per unit x, y area, dF_{x1}/dA_{x1} , in the lateral direction and the force per unit x, y area, dF_{z1}/dA_{x1} , in the normal direction. The dot product of the r and n unit vectors is Eq. 13 and the conversion of the area differential from the s, y plane to the x, y plane is accomplished through Eq. 14. One needs to now find the partial derivatives of r and n with respect to x and z . Using Eq. 15 and then referring to Fig. 6 one obtains:

$$\begin{aligned} \frac{\partial r_1}{\partial x_1} &= \frac{-(X_{2r} - x_1)}{r_1} = \sin \theta_2 & \frac{\partial r_1}{\partial z_1} &= \frac{-(Z_2(X_{2r}) - Z_1(x_1))}{r_1} = -\cos \theta_2 \\ \frac{\partial n_1}{\partial x_1} &= \frac{-(X_{2n} - x_1)}{n_1} = \sin \theta_1 & \frac{\partial n_1}{\partial z_1} &= \frac{-(Z_2(X_{2n}) - Z_1(x_1))}{n_1} = -\cos \theta_1 \\ \frac{\partial r_2}{\partial x_2} &= \frac{(x_2 - X_{1r})}{r_2} = -\sin \theta_1 & \frac{\partial r_2}{\partial z_2} &= \frac{(Z_2(x_2) - Z_1(X_{1r}))}{r_2} = \cos \theta_1 \\ \frac{\partial n_2}{\partial x_2} &= \frac{(x_2 - X_{1n})}{n_2} = -\sin \theta_2 & \frac{\partial n_2}{\partial z_2} &= \frac{(Z_2(x_2) - Z_1(X_{1n}))}{n_2} = \cos \theta_2 \end{aligned} \quad (18)$$

By use of appropriate trigonometric identities and again observing that $dZ_1/dx_1 = \tan\theta_1$ and $dZ_2/dx_2 = \tan\theta_2$ one obtains:

$$\begin{aligned}\cos\theta_1 &= \frac{1}{\sqrt{\left(\frac{dz_1}{dx_1}\right)^2 + 1}} & \cos\theta_2 &= \frac{1}{\sqrt{\left(\frac{dz_2}{dx_2}\right)^2 + 1}} \\ \sin\theta_1 &= \frac{\frac{dz_1}{dx_1}}{\sqrt{\left(\frac{dz_1}{dx_1}\right)^2 + 1}} & \sin\theta_2 &= \frac{\frac{dz_2}{dx_2}}{\sqrt{\left(\frac{dz_2}{dx_2}\right)^2 + 1}}\end{aligned}\quad (19)$$

4.4. Comparing PFA force calculations with extended PFA for a set of corrugated flat plates

4.4.1. Normal forces

It is desirable to compare the calculated normal forces from the PFA in Eq. 6 and 7 with the calculated normal forces from the extended PFA using Eq. 17-19. The appropriate Z_1 and Z_2 functions for a set of corrugated flat plates can be obtained from Eq. 10 describing the corrugated sphere and corrugated flat plate by substituting z_0 for z_{sph} . Other variables are as indicated in Fig. 3. The values used in the calculations are those used in the Chen / Mohideen experiment shown in nomenclature with the added stipulation that phase angle $\phi = 0$. Calculated normal force per unit x, y area obtained by using the PFA is compared in Fig. 7 with the normal force per unit x, y area on the Z_1 and Z_2 plates obtained by using the extended PFA. For the extended PFA Fig. 7 also shows the difference in magnitude of normal force per unit x, y area between the Z_1 and Z_2 plates. The difference in magnitude of normal force per unit area for the regular PFA is zero as was mentioned previously.

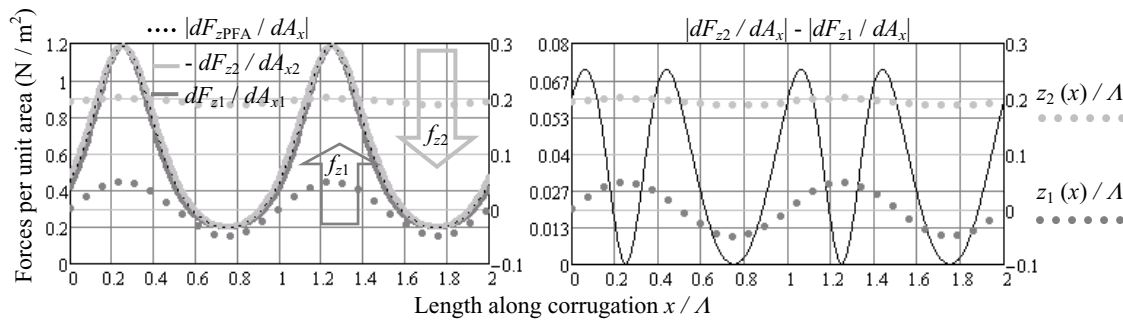


Fig. 7. Comparing normal force per unit area calculated using PFA with extended PFA on Z_1 and Z_2 plates for a corrugated set of flat plates

The extended PFA compares well with the regular PFA except for the small difference in the normal force per unit area between the Z_1 and Z_2 plates. To be noted, the normal force per unit x, y area difference is greater than or equal to zero for an entire corrugated wavelength. Also it is greatest when the difference in slopes of Z_1 and Z_2 are greatest and zero when the slopes are zero. The net force per unit area when it is non-zero is in the negative direction – from the plate having the smaller corrugation amplitude to that having the larger.

In Fig. 8 the normal force per unit x, y area is integrated over the area of the Z_1 and Z_2 plates, which for convenience have been set to 1 cm x 1 cm. The total normal forces obtained by the PFA and the extended PFA for each of the Z_1 and Z_2 plates is then plotted against the phase angle between the corrugations in the Z_1 and Z_2 plates. The normal force differences between Z_1 and Z_2 plates are also plotted for the extended PFA. The normal force differences for the regular PFA are zero.

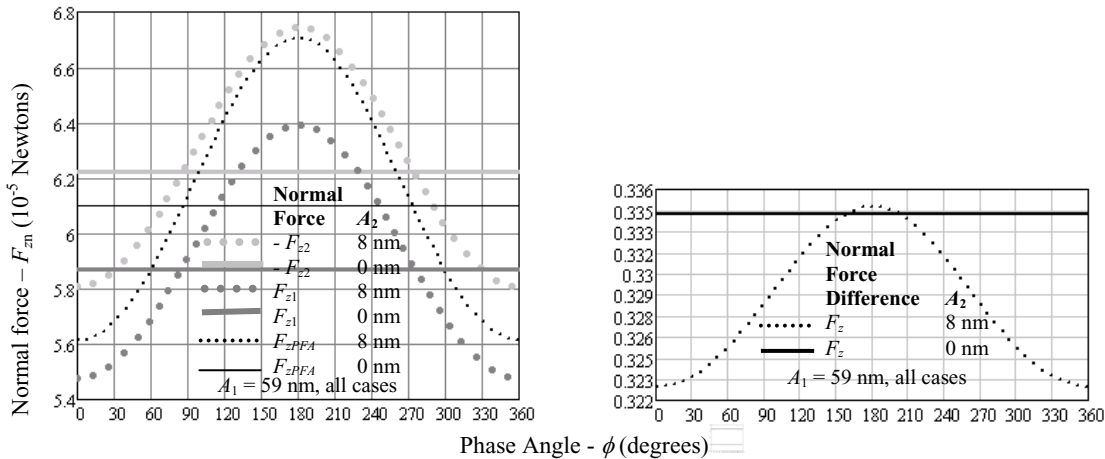


Fig.8. Total normal force on 1 cm x 1 cm plates using PFA and extended PFA versus phase angle

4.4.2 Lateral forces

It is desirable to compare the calculated lateral forces from the PFA in Eq. 8 and 9 with the calculated lateral forces from the extended PFA using Eq. 17-19. The appropriate Z_1 and Z_2 functions for a set of corrugated flat plates can be obtained from Eq. 10 describing the corrugated sphere and corrugated flat plate by substituting z_0 for z_{sph} . Other variables are as indicated in Fig. 3. The values used in the calculations are those used in the Chen / Mohideen experiment shown in nomenclature with the added stipulation that phase angle $\phi = 0$. Calculated lateral force per unit x, y area obtained by using the PFA is compared in Fig. 9 with the lateral force per unit x, y area on the Z_1 and Z_2 plates obtained by using the extended PFA. For the extended PFA Fig. 9 also shows the difference in the lateral force per unit x, y area between the Z_1 and Z_2 plates. The difference in the lateral force per unit area for the regular PFA is not zero as was the case for the normal forces per unit area but alternates between positive and negative.

In Fig. 10 the lateral force per unit x, y area is integrated over the area of the Z_1 and Z_2 plates, which for convenience have been set to 1 cm x 1 cm. The total lateral forces obtained by the PFA and the extended PFA for each of the Z_1 and Z_2 plates is then plotted against the phase angle between the corrugations in the Z_1 and Z_2 plates. The lateral force differences between Z_1 and Z_2 plates are also plotted for the extended and regular PFA. The lateral force differences for the regular PFA are zero.

Though the lateral force and force difference per unit area look very different for the regular and extended PFA, the total lateral forces on the plates versus phase angle assuming various corrugation amplitudes are very similar. Also to be noted, the extended PFA had a force difference when the corrugation amplitudes were different on the two plates whereas the regular PFA did not. For cases where the corrugation amplitude was zero for one of the plates or was the same as the corrugation amplitude of the other plate the extended PFA like the regular PFA showed no lateral force difference.

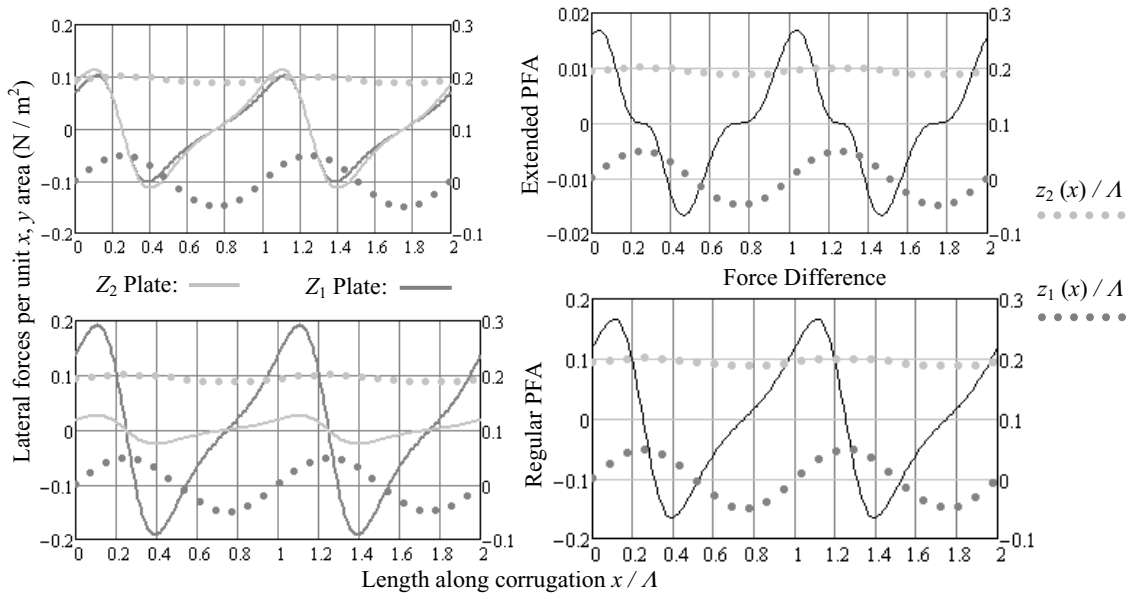


Fig. 9. Lateral force and force difference per unit area versus length along corrugation for PFA and extended PFA

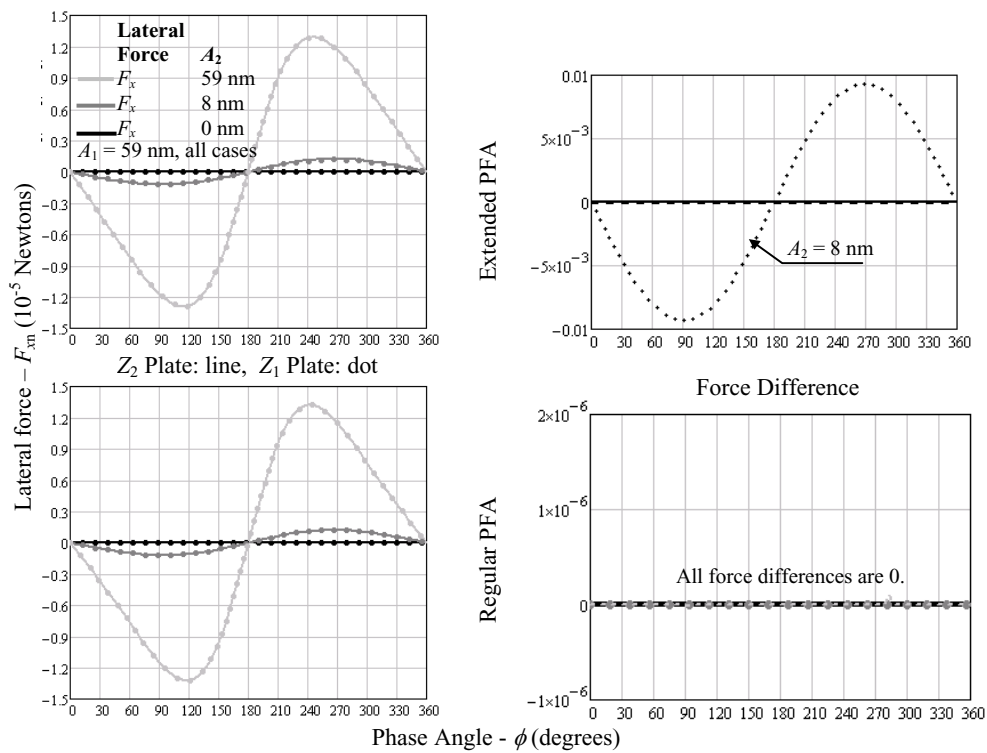


Fig. 10. Total lateral force on 1 cm x 1 cm plates using PFA and extended PFA versus phase angle

4.5. Constraints on extended PFA

The regular PFA only holds when the separation distance is much smaller than the length scales characterized by the surface geometry such as the radius of curvature for spherical and cylindrical plates and corrugation wavelength for corrugated surfaces. The extended PFA has similar constraints. As can be seen by a re-examination of Fig. 4 the radius of curvature of plate curves that are concave relative to the Casimir cavity must be larger than the distances between plates r_1 , n_1 , r_2 , n_2 in order for those distances to be unambiguous meaning that there is one and only one shortest distance between a point on one plate and the opposite plate. For corrugated plates for example the corrugation wavelength must be much longer than the nominal distance between plates as with the regular PFA.

5. Conceptual experiment testing conservative forces

Two conceptual experiments are being initially proposed to test whether asymmetrically corrugated plates have conservative or non-conservative forces between them. The first experiment tests whether there exists a physical phenomenon. The second controls against external sources of energy. While corrections for plasma wavelength haven't been included in the calculations comparing the regular with the extended PFA since such corrections would be nearly the same with both sets of calculations. These corrections would be required in order to predict expected experimental results.

5.1. A physical phenomenon?

The first experiment consists of a pinwheel like arrangement made up of two or four pairs of plates where one of the plates making up the pair is corrugated and the other is flat. According to Figs. 9 and 10 the normal force points in the direction of the flat to corrugated plate. The pairs of plates would be arranged in such fashion so that the pinwheel would rotate. The pinwheel apparatus in turn is placed in a vacuum. The Casimir cavity is maintained by there being ridges in the flat plate running perpendicular to the corrugations. The ridges need to be far enough apart so that the corrugations in the opposite plate dominate the interaction and yet close enough so that Casimir forces don't unduly deform the plates. The dimensions of the corrugations and the distance between plates within the Casimir cavities should be that of a regime that has already been tested experimentally such as that of the Chen / Mohideen experiment. If a physical phenomenon is detected it will imply that the two interpretations for the source of the Casimir effect are not equivalent.

5.2. Origin of forces

The second experiment is performed if the first experiment is successful. It tests whether the forces originate from between the plates or externally such as a radiometer. To perform this test at least three pairs of plates are required, preferably four. The pairs of plates are placed one on top of the other and then weighed on a scale of sufficient sensitivity. The forces of the plate pairs are first orientated in one direction. With subsequent weighings a plate pair is flipped to the opposite direction so that eventually all plate pairs are pointed in the opposite direction. With each pair flip there should be an incremental increase (or decrease) in the weight of the aggregate set of plate pairs. If energy is coming from the outside presumably only the exposed plate pair would influence the weight of the aggregate.

Both of these experiments are conceptually very simple. The main problem will be that of quality control – making sure that the variables one thinks one is measuring are actually the variables that are being measured.

6. Conclusion

This paper has presented a speculative but plausible model for an extension of the popular PFA that is premised upon vacuum fields that are dynamic and perhaps steady state but not static. This extended PFA model fixes some minor problems with the regular PFA and is in a sense, because it incorporates local geometry, more natural. The down side is that it produces non-conservative forces, which may be an artifact of the model. On the other hand, it has been shown that the trivial, and one could say artificial, conservativeness of the PFA is an artifact of its ignoring local geometry. Vacuum centric models may in fact be conservative but not trivially so.

As opposed to the vacuum centric PFA, bulk matter centric models such as PWS are inherently conservative. If vacuum centric and bulk matter centric models are truly equivalent then vacuum centric models would need to be conservative because of this equivalence. However, in the case of PWS at least, there have been found some reasons to question this equivalence. These reasons being: 1) The PWS calculation of forces between parallel plates (without normalization or calibration) is much less than the classical Casimir calculation of forces, 2) screening of more distant atoms by closer atoms in the bulk matter calculation makes that difference even greater and 3) the fact that the zero-point vacuum interpretation doesn't involve the plate thickness in its force calculation and corrections whereas the plate thickness is inherent in the bulk matter interpretation. These issues don't prove that the Casimir effect involving asymmetric geometries can lead to non-conservative forces but perhaps begin to raise the question whether they can. Perhaps they also begin to provide motivation to do an experiment that can resolve the issue.

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